<u>SM315 Supplementary Problems #2</u>
General Heat Equation Problems Requiring Fourier Series Solutions

Find a formal solution to the given initial-boundary value problems. Use computer aids as necessary to determine coefficients resulting from Fourier series analysis. You may also use Table 2.4.1 on Page 69 to reduce the work (This presumes that you have done enough separation of variable problems to skip the steps that lead to these solutions)

1. $\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{dx^2}, \quad 0 < x < \pi, \quad t > 0 \\ u(0,t) = u(\pi,t) = 0, \quad t > 0 \\ u(x,0) = x^2, \quad 0 < x < \pi \end{cases}$	2. $\begin{cases} \dfrac{\partial u}{\partial t} = 5\dfrac{\partial^2 u}{dx^2}, \quad 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t) = 0, \quad t > 0 \\ u(x,0) = (1-x)x^2, \quad 0 < x < 1 \end{cases}$
3. $\begin{cases} \dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{dx^2}, \quad 0 < x < 1 \quad t > 0 \\ \dfrac{\partial u}{\partial x}(0,t) = \dfrac{\partial u}{\partial x}(1,t) = 0, \quad t > 0 \\ u(x,0) = (1-x)x^2, \quad 0 < x < 1 \end{cases}$	4. $\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{dx^2}, \quad 0 < x < \pi \quad t > 0 \\ u(0,t) = 0, \quad u(\pi,t) + \dfrac{\partial u}{\partial x}(\pi,t) = 0, \quad t > 0 \\ u(x,0) = f(x), \quad 0 < x < \pi \end{cases}$
5. $\begin{cases} \dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{dx^2} + x, \quad 0 < x < \pi \quad t > 0 \\ u(0,t) = u(\pi,t) = 0, \quad t > 0 \\ u(x,0) = \sin(x), \quad 0 < x < \pi \end{cases}$	6. $\begin{cases} \dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{dx^2} + 5, \quad 0 < x < \pi \quad t > 0 \\ u(0,t) = 0, \quad u(\pi,t) = 1, \quad t > 0 \\ u(x,0) = 1, \quad 0 < x < \pi \end{cases}$