SM315 Lecture Notes
Introduction, Classification of PDE's
Homework: (Handout)

Top Board

Classifying DE's/PDE's

- **Order**: Highest derivative/partial derivative
- **Dependent Variable**: Appear in top of derivative expression.
- **Independent Variable**: Appear in bottom of derivative expression.
- **Kind of Coefficients**: Constant vs. Variable
- **Linear/Non-Linear**: Dependent Variable appears as Linear.
- **DE**-On Independent Variable
- **PDE**-More than one Independent Variables
- **Second Order Linear DE in 2 Variables**: $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$
  - **Parabolic**: $B^2 - 4AC = 0$
  - **Elliptic**: $B^2 - 4AC < 0$
  - **Hyperbolic**: $B^2 - 4AC > 0$

1. Recall: Classification of Ordinary Differential Equations

a. *Example (falling body):* $m\frac{d^2x}{dt^2} = mg \rightarrow \frac{d^2x}{dt^2} = g$

i. **Order: 2nd Order DE in 1 variable**

ii. **Dependent/Independent Variable:** $x(t)$

iii. **Homogeneous/Non-Homogenous: Non-Homogenous**

iv. **Variable/Constant Coefficients: Constant**

v. **Linear/Non-Linear: Linear**

b. ***Example (mass-spring system):*** $m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=0$

i. **Order: 2nd Order DE in 1 variable**

ii. **Dependent/Independent Variable:** $y(t)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Constant**

v. **Linear/Non-Linear: Linear**

2. Classification of Ordinary Partial Differential Equations

a. ***Example (1-D Wave Equation):*** $u_{tt}=u_{xx}\rightarrow u_{tt}-u_{xx}=0$

i. **Order: 2nd Order DE in 2 variables**

ii. **Dependent/Independent Variable:** $u(x,t)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Constant**

v. **Linear/Non-Linear: Linear**

vi. **Since:** $A=1, B=0, C=-1\rightarrow B^2-4AC=4>0\rightarrow$ **Hyperbolic**

1. Note designation can only be made if PDE has 2 independent variables, is linear, and has constant coefficients A, B, C.
2. If coefficient is variable, characterization may change.

b. ***Example:*** $uu_{tt}=u_{xx}\rightarrow uu_{tt}-u_{xx}=0$

i. **Order: 2nd Order DE in 2 variables**

ii. **Dependent/Independent Variable:** $u(x,t)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Constant**

v. **Linear/Non-Linear: Non-Linear**

vi. **Since Non-Linear can not make further designation**

c. ***Example:*** $tu_{tt} = u_{xx} \to tu_{tt} - u_{xx} = 0$

i. **Order: 2^{nd} Order DE in 2 variables**

ii. **Dependent/Independent Variable:** $u(x,t)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Variable**

v. **Linear/Non-Linear: Linear**

vi. **Since:** $A = t, B = 0, C = -1 \to B^2 - 4AC = 4t > 0 \to$ **Hyperbolic**

1. Assumes $t > 0$

d. ***Example:*** $xu_{tt} = u_{xx} \to tu_{tt} - u_{xx} = 0$

i. **Order: 2^{nd} Order DE in 2 variables**

ii. **Dependent/Independent Variable:** $u(x,t)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Variable**

v. **Linear/Non-Linear: Linear**

vi. **Since:** $A = x, B = 0, C = -1 \to B^2 - 4AC = 4x > 0 \to$ **Hyperbolic**

1. $x > 0 \to hyperbolic$
2. $x = 0 \to parabolic$
3. $x < 0 \to elliptic$

e. ***Example (3D Laplacian):*** $\nabla^2 u = 0 \to u_{xx} + u_{yy} + u_{zz} = 0$

i. **Order: 2^{nd} Order DE in 3 variables**

ii. **Dependent/Independent Variable:** $u(x, y, z)$

iii. **Homogeneous/Non-Homogenous: Homogenous**

iv. **Variable/Constant Coefficients: Constant**

v. **Linear/Non-Linear: Linear**

1. Since more than two independent variables, we make no further characterization.

3. Group Work-Homework Handout