SM315 Lecture Notes
Laplace's Equation on a Rectangle Example

1. Results form Day 21 Lecture

a. $$\begin{cases} PDE: & u_{xx} + u_{yy} = 0 \\ BC1: & u(0,y) = g(y) \\ BC2: & u(L,y) = 0 \\ BC2: & u(x,0) = 0 \\ BC2: & u(x,H) = 0 \end{cases}$$

b. $$\begin{cases} u(x,y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{H} y\right) \sinh\left(\frac{n\pi}{H}(x-L)\right) \\ A_n = \frac{2}{H \sinh(-Ln\pi/H)} \int_0^H \left(g(y) \sin\left(\frac{n\pi}{H} y\right)\right) dy \end{cases}$$

2. Example Problem

a. $$\begin{cases} PDE: & u_{xx} + u_{yy} = 0 \\ BC1: & u(0,y) = y(y-1) \\ BC2: & u(1,y) = 0 \\ BC2: & u(x,0) = 0 \\ BC2: & u(x,1) = 0 \end{cases}$$ →i.e. $L = 1,\ H = 1,\ g(y) = y(y-1)$

b. $$\begin{cases} u(x,y) = \sum_{n=1}^{\infty} A_n \sin(n\pi y) \sinh(n\pi(x-1)) \\ A_n = \frac{2}{\sinh(-n\pi)} \int_0^1 (y(y-1)\sin(n\pi y)) dy \\ \rightarrow A_n = \frac{4}{n^3 \pi^3 \sinh(n\pi)} \left((-1)^{n+1} + 1\right) \end{cases}$$

3. Maple Demo Problem

4. What if we Have Boundary Conditions on All Four Sides

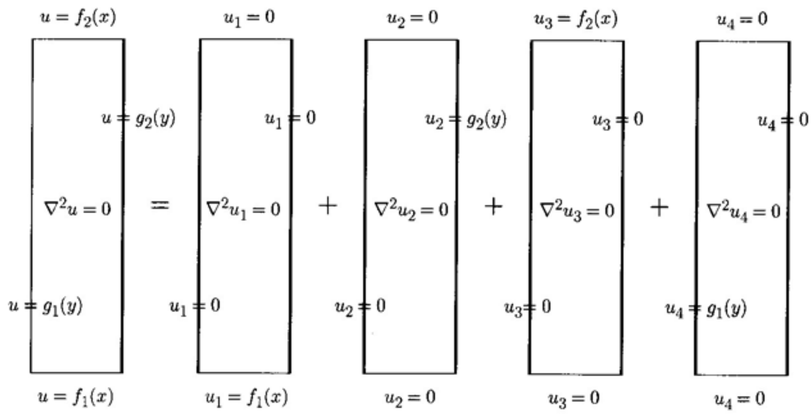

Figure 2.5.1 Laplace's equation inside a rectangle.

a. Discuss this strategy.
b. Handout project.
c. Email Demo